

Brief Notice / Kurze Mitteilung

Simplification of the Sampling Variance of the Correlation Coefficients

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Abe (1969) developed the sampling variance of the genetic correlation estimated from analysis of variance and covariance based on a two-stage nested classification. Grossman (1970) generalized this to all correlation coefficients (genetic, environmental, and phenotypic). Hammond and Nicholas (1972), using Grossman's method, dealt with unbalanced data and corrected errors in Grossman's equation 13.

To minimize the probability of errors in arithmetic, we have simplified Hammond and Nicholas' equation 10. In Grossman's notation, it is:

$$\begin{aligned} \text{Est. Var}(\hat{r}_\theta) = & \frac{f^2 \hat{r}_\theta^2}{(nd)^2} \left\{ \frac{a^2}{2u} \left(\frac{2U_{11}U_{22}}{\hat{\theta}_{12}^2} + \frac{2U_{12}^2}{\hat{\theta}_{12}^2} + \frac{U_{11}^2}{\hat{\theta}_{11}^2} \right. \right. \\ & + \frac{U_{22}^2}{\hat{\theta}_{22}^2} - \frac{4U_{11}U_{12}}{\hat{\theta}_{11}\hat{\theta}_{12}} - \frac{4U_{12}U_{22}}{\hat{\theta}_{12}\hat{\theta}_{22}} + \frac{2U_{12}^2}{\hat{\theta}_{11}\hat{\theta}_{22}} \Big) \\ & + \frac{b^2}{2v} \left(\frac{2V_{11}V_{22}}{\hat{\theta}_{12}^2} + \frac{2V_{12}^2}{\hat{\theta}_{12}^2} + \frac{V_{11}^2}{\hat{\theta}_{11}^2} + \frac{V_{22}^2}{\hat{\theta}_{22}^2} \right. \\ & - \frac{4V_{11}V_{12}}{\hat{\theta}_{11}\hat{\theta}_{12}} - \frac{4V_{12}V_{22}}{\hat{\theta}_{12}\hat{\theta}_{22}} + \frac{2V_{12}^2}{\hat{\theta}_{11}\hat{\theta}_{22}} \Big) + \frac{c^2}{2w} \\ & \times \left(\frac{2W_{11}W_{22}}{\hat{\theta}_{12}^2} + \frac{2W_{12}^2}{\hat{\theta}_{12}^2} + \frac{W_{11}^2}{\hat{\theta}_{11}^2} + \frac{W_{22}^2}{\hat{\theta}_{22}^2} \right. \\ & \left. \left. - \frac{4W_{11}W_{12}}{\hat{\theta}_{11}\hat{\theta}_{12}} - \frac{4W_{12}W_{22}}{\hat{\theta}_{12}\hat{\theta}_{22}} + \frac{2W_{12}^2}{\hat{\theta}_{11}\hat{\theta}_{22}} \right) \right\}. \end{aligned}$$

Rearrangement leads to:

$$\begin{aligned} \text{Est. Var}(\hat{r}_\theta) = & \frac{f^2 \hat{r}_\theta^2}{2(nd)^2} \left\{ \frac{a^2}{u} \left[\left(\frac{U_{11}}{\hat{\theta}_{11}} - \frac{U_{12}}{\hat{\theta}_{12}} \right)^2 \right. \right. \\ & + \left(\frac{U_{22}}{\hat{\theta}_{22}} - \frac{U_{12}}{\hat{\theta}_{12}} \right)^2 + 2 \left(\frac{U_{12}}{\hat{\theta}_{11}} - \frac{U_{22}}{\hat{\theta}_{12}} \right) \\ & \times \left(\frac{U_{12}}{\hat{\theta}_{22}} - \frac{U_{11}}{\hat{\theta}_{12}} \right) \Big] + \frac{b^2}{v} \left[\left(\frac{V_{11}}{\hat{\theta}_{11}} - \frac{V_{12}}{\hat{\theta}_{12}} \right)^2 \right. \\ & + \left(\frac{V_{22}}{\hat{\theta}_{22}} - \frac{V_{12}}{\hat{\theta}_{12}} \right)^2 + 2 \left(\frac{V_{12}}{\hat{\theta}_{11}} - \frac{V_{22}}{\hat{\theta}_{12}} \right) \\ & \times \left(\frac{V_{12}}{\hat{\theta}_{22}} - \frac{V_{11}}{\hat{\theta}_{12}} \right) \Big] + \frac{c^2}{w} \left[\left(\frac{W_{11}}{\hat{\theta}_{11}} - \frac{W_{12}}{\hat{\theta}_{12}} \right)^2 \right. \\ & + \left(\frac{W_{22}}{\hat{\theta}_{22}} - \frac{W_{12}}{\hat{\theta}_{12}} \right)^2 + 2 \left(\frac{W_{12}}{\hat{\theta}_{11}} - \frac{W_{22}}{\hat{\theta}_{12}} \right) \\ & \left. \left. \times \left(\frac{W_{12}}{\hat{\theta}_{22}} - \frac{W_{11}}{\hat{\theta}_{12}} \right) \right] \right\}. \end{aligned}$$

Letting:

$$\begin{aligned} U &= \begin{bmatrix} U_{11} & U_{12} \\ U_{12} & U_{22} \end{bmatrix}, & V &= \begin{bmatrix} V_{11} & V_{12} \\ V_{12} & V_{22} \end{bmatrix}, \\ W &= \begin{bmatrix} W_{11} & W_{12} \\ W_{12} & W_{22} \end{bmatrix}, & \text{and } \theta &= \begin{bmatrix} \hat{\theta}_{11}^{-1} & -\hat{\theta}_{12}^{-1} \\ -\hat{\theta}_{12}^{-1} & \hat{\theta}_{22}^{-1} \end{bmatrix}, \end{aligned}$$

we have:

$$\text{Est. Var}(\hat{r}_\theta) = \frac{f^2 \hat{r}_\theta^2}{2(nd)^2} \text{tr} \left\{ \frac{a^2}{u} (U\theta)^2 + \frac{b^2}{v} (V\theta)^2 + \frac{c^2}{w} (W\theta)^2 \right\}.$$

Thus the arithmetic is markedly systemized and reduced in detail, and is more simply and compactly programmable.

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The need for this simplification was suggested by Dr. A. W. Nordskog, Iowa State University.

Literature

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